Engineering Metrology
MNF 325

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Department of Manufacturing Engineering and Production
Course Outline

Lecturer: Prof. Dr. Nabil Gadallah


Course Outline:
1. Introduction to concept of metrology and quality
2. History and units of measurement
3. Analysis of errors and uncertainties in measurement
4. Linear and angular measurements
5. Advanced measurement techniques (optical, laser, ultrasound, etc.)
6. Measurement of geometric shape (roundness, flatness, etc.)
7. Measurement of surface texture (surface roughness and waviness)
8. Measurement of screw threads
9. Measurement of gears
10. Limits and limit gauges

Exams, Projects, Practices:
• Workshop and/or classroom practices (10%)
• Midterm Exam (10%)
• Practical Exam (20)
• Term Exam (60%)

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CHAPTER (1)
Introduction to Metrology and Quality
Definition of Metrology and Measurement

Metrology

- from Greek “metron” (measure) and –logy.
- the science of measurement.
- includes all theoretical and practical aspects of measurement.

Measurement

- Comparison of a specimen (a thing or a quantity) to a standard, within some agreement upon reference frame.
- A successful measurement must maintain (i.e. apply controls to) specimen, standard, and a stable reference frame.
- All measurements based on standard.
- Maintenance of standards & reference frame critical to any measurement.
Objective Classification of Measuring Technological Concept

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Product Quality

- a product’s fitness for use.
- the totality of features that bear on a product’s ability to satisfy a given need.

Need for Quality

- Quality is a very important aspect of manufacturing.
- It is a big issue (consider TQM, Six Sigma, Taguchi, ISO Standards, etc.)
- Needed for interchangeable manufacturing.
- Basic concept of standardization and mass production.
- Components of a product must fit together, assemble properly and be replaceable.
- Quality should be built into a product.
- Prevention of defects is a major goal.
Measure of Quality

High Quality Product

☺ performs its functions reliably
☺ performs its functions for a long time
☺ performs its functions conveniently

Low Quality Product

◎ does not perform its function reliably
◎ fails or breaks after short time of use
◎ difficult to use

GOAL
Continuous Quality Improvement
(functionality, reliability, durability, …)

Inspection (Measurement)

What?  When?  How?
What to Inspect?

**Inspection specific to PRODUCTS**

- Electronic parts (circuits, chips, etc.)
- Machine elements (engines, brakes, gears, etc.)
- Heat and thermodynamic components (engines, fuel injectors, etc.)
- Medical and Bio-related products (implants, dental devices, surgical parts, etc.)
- Aerospace components (turbine blades and discs, airplane body, etc.)

**Inspection specific to PROCESSES**

- Chip removal processes (turning, milling, drilling, etc.)
- Chipless manufacturing (casting, molding, forging, etc.)
- Non-traditional methods (EDM, ECM, ultrasonics, etc.)
When to Inspect?

**Inspection AFTER production**

- x costly production steps already complete
- x high cost of rejection or rework
- x difficult to test for all possible defects
- x difficult to identify responsibility for defect

**Inspection DURING production**

- ✓ defects found early, at each production step
- ✓ reduced cost of rejection or rework
- ✓ facilitates continuous process improvement
Measurement of DIMENSIONS
• Linear measurements (length, thickness, etc.)
• Angular measurements (taper, angle, etc.)
• Measurement of surface texture (roughness, waviness, etc.)
• Measurement of geometric shape (roundness, flatness, squareness, etc.)
• Measurement of screw threads and gears …

Inspection for DIMENSIONAL ACCURACY
• post-process (traditional)
• in-process (modern trend)

DIMENSIONAL TOLERANCES
• permissible variation in dimensions
• directly affects product quality and cost
Scale of Measurement

Large scale (low frequency) measurements
- Measurements at macro levels
- Dimension (length, angle, etc.)
- Tolerance
- Form error (contour measurement)

Medium scale (medium frequency) measurements
- Measurements at meso levels
- Surface texture/topography (waviness)
- Geometric shape (flatness, roundness, etc.)

Small scale (high frequency) measurements
- Measurements at micro levels
- Surface texture/topography (surface roughness)
Engineering Metrology

CHAPTER (2)

History and Units of Measurement
Measurement and Mathematics

> Measurement constitutes the first steps towards mathematics.

> In other words, associating numbers with physical objects...

> Thus, compare the objects by comparing the associated numbers...

1 FOOT FOR LENGTH

SEEDS AND BEANS FOR WEIGHT
Brief History of Measurement

Egyptians (around 3000 BC)
> Earliest known measurement was **Egyptian cubit**.

Babylonians (around 1700 BC)
> Their weights and measures had a wider influence.
> Their system was approximately same with Egyptians.
> However, there were small differences about lengths.

In summary:
> European system based on Roman measures
> Roman system based on Greek measures
> Greek system based on breadth of finger

Harappans (between 2500 BC and 1700 BC)
> They had flourished in Punjab (in India).
> They adopted a uniform system of weights and measures.
> Several scales for the measurement of length were also discovered during excavations.

**Current measurement systems have been derived from 3000 BC.**
Anthropic Measurements and Units

Digit: Egyptians - breadth of forefinger
Inch: breadth of thumb
Palm: breadth of four fingers
Span: tip of thumb to tip of little finger (hand spread)
Cubit: Egyptians - elbow to tip of middle finger
Foot: length of man's foot
Yard: King Henry I of England - tip of his nose to end of thumb
Mile: 5000 Roman feet
Pace: Romans - a full stride

and many others...
Early Measurements

But there was a problem!

There were a lot of different lengths which change from human to human.

Solution was to develop a standard measurement: a black granite rod !!!

There were 28 digits in a cubit, 4 digits in a palm, 5 digits in a hand, 3 palms (so 12 digits) in a small span, 14 digits (or a half cubit) in a large span, 24 digits in a small cubit, and so on...

But a problem again: How could they measure a measurement smaller than 1 digit?

For this, the Egyptians used measures composed of unit fractions...

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System of Units

SI System

> **Le systeme International d’unites** or **International System of Units**
> Also called Metric System (i.e. all measurements based on m, kg, s).
> Seven **base** units (m, kg, s, A, °K, mol, cd) and other **derived** units.
> Used throughout the world except USA and some part of UK.

English (British or Imperial) System

> Various measurements based on **different units**: 
> Length: inches / feet / yards / miles / ...
> Mass: ounces / pounds / tones / ...
> Volume: fluid ounce / cups / quarts / gallons / ...

Why do we prefer SI system?

> SI units **work in 10s**, so it is **very easy to convert** big units to small units, or vice versa (e.g. 1 m = 100 cm). However, English units are **not easy to convert**. Can you easily convert mile to inches?
> In SI system, a **unique unit** is used for a quantity (e.g. the unit of distance is meter). However, **various units** may be used in English system (e.g. for distance: inch, foot, yard, mile, and so on.)
Chronological History of Metric System

1670  **Gabriel Moulton**, a French mathematician, proposes a measurement system based on a physical quantity of nature and not on human anatomy.

1790  **The French Academy of Science** recommends the adoption of a system with a unit of length equal to one ten-millionth of the distance on a meridian between the Earth’s North Pole and equator.

1870  A French conference is set up to work out standards for a unified metric system.

1875  **The Treaty of Meter** is signed by 17 nations (including United States). This has established a permanent body with the authority to set standards.

1893  The United States officially adopts the metric system standards as bases for weights and measures (but continues to use British units).

1975  **The Metric Conversion Act** is enacted by Congress. It states “The policy of the United States shall be to coordinate and plan the increasing use of the metric system in the United States and to establish a United States Metric Board to coordinate the voluntary conversion to the metric system.” (This means that no mandatory requirements are made.)
SI Base Units - Meter

Definition

> **Meter (m)** is the unit of **length**.

> It was originally defined as a unit equal to one ten-millionth part of a quadrant of Earth's meridian as measured from North Pole to Equator.

> It is now precisely equal to the length of path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.

International Prototype

> **International Prototype Metre bar**, made of an alloy of platinum and iridium, was the standard from 1889 to 1960.

> It is kept at **Bureau International des Poids et Measures** (International Bureau of Weights and Measures) near Paris.
SI Base Units - Kilogram

**Definition**

> **Kilogram (kg)** is the unit of mass.

> Originally, it was defined as the mass of a volume of pure water equal to a cube of one tenth meter at the temperature of melting ice.

> It is now equal to the mass of international prototype of kilogram.

**International Prototype**

> There are currently two prototypes: the one kept at Bureau International des Poids et Mesures (BIPM) in Paris, and the other one at National Institute of Standards and Technology (NIST) in Washington D.C.

> **The prototype at BIPM** is made of a platinum alloy known as “Pt-10Ir”, which is 90% platinum and 10% iridium (by mass).

> It was machined into a right-circular cylinder with equal height and diameter (39.17 mm) to minimize its surface area.

> **The prototype at NIST** is actually 0.999 999 961 kg of the one at BIPM.
Mass vs Weight

> **Mass** is *stuff* (a quantity of matter) regardless of what it is made of, any number of other conditions (including shape, temperature, color), where it is, and how it moves (disregarding relativistic effects).

> **Weight** is *force* (specifically gravitational force) acting on the stuff in a special way. It is the force necessary to cause the stuff to accelerate at gravitational acceleration of 9.81 m/s² (32.174 ft/s²).

mass *(kg or lb)* remains constant

but

weight *(kgf or lbf)* varies

1 kg of cotton and 1 kg of iron are **EQUAL** in mass

**(NOT EQUAL** in weight)
Definition

- **Second (s)** is the unit of **time**.

- It was originally defined as 1/86 400 of a solar day.

- Today, it is equal to the duration of 9 192 631 770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of cesium-133 atom.

- The frequency of a specific transition between two of cesium's energy levels can be **accurately monitored**, and thus it is used to define the second.
SI Base Units – Ampere, Kelvin, Mole, Candela

Ampere

> **Ampere (A)** is the unit of **electric current**.

> Being maintained in two straight parallel conductors of infinite length with negligible circular cross section and placed 1 m apart in vacuum, it is equal to that constant current which would produce a force (between these conductors) equal to $2 \times 10^{-7}$ Newton per meter of length.

Kelvin

> **Kelvin (°K)** is the unit of **thermodynamic temperature**.

> It is equal to the fraction of thermodynamic temperature of triple point of water (i.e. the point where water exists in gaseous, liquid and solid phases simultaneously) at 273.16 °K (0.01 °C, 32.02 °F).

Mole

> **Mole (mol)** is the unit of **the amount of substance of a system**.

> It is equal to the amount of substance of a system which contains as many elementary entities (i.e. atoms, molecules, ions, electrons) as there are atoms in 0.012 kilogram of carbon-12.

Candela

> **Candela (cd)** is the unit of **luminous intensity**.

> It is equal to luminous intensity of a source in a given direction that emits monochromatic radiation of frequency (540 x 1012 hertz) and that has a radiant intensity of 1/683 watt per steradian.
SI Coherent Derived Units with Special Names and Symbols

- kilogram (kg) - MASS
- meter (m) - LENGTH
- second (s) - TIME
- mole (mol) - AMOUNT OF SUBSTANCE
- ampere (A) - ELECTRIC CURRENT
- kelvin (K) - THERMODYNAMIC TEMPERATURE
- candela (cd) - LUMINOUS INTENSITY

- newton (N) - FORCE
- joule (J) - ENERGY, WORK, QUANTITY OF HEAT
- watt (W) - POWER, HEAT FLOW RATE
- steradian (sr) - SOLID ANGLE
- radian (rad) - PLANE ANGLE

- pascal (Pa) - PRESSURE, STRESS
- gray (Gy) - ABSORBED DOSE
- sievert (Sv) - DOSE EQUIVALENT

- gray (Gy) - ABSORBED DOSE
- sievert (Sv) - DOSE EQUIVALENT

- hertz (Hz) - FREQUENCY

- becquerel (Bq) - ACTIVITY (OF A RADIONUCLIDE)

- katal (kat) - CATALYTIC ACTIVITY

- weber (Wb) - MAGNETIC FLUX
- tesla (T) - MAGNETIC FLUX DENSITY

- henry (H) - INDUCTANCE
- volt (V) - VOLTAGE, ELECTROMOTIVE FORCE

- coulomb (C) - ELECTRIC CHARGE

- kelvin (K) - CELSIUS TEMPERATURE
  t/°C = t/K - 273.15

- farad (F) - CAPACITANCE

- ohm (Ω) - RESISTANCE
- siemens (S) - CONDUCTANCE
## SI Prefixes (Multipliers)

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>$10^n$</th>
<th>Multiplier</th>
</tr>
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<tr>
<td>yotta</td>
<td>Y</td>
<td>$10^{24}$</td>
<td>$1 000 000 000 000 000 000 000 000$</td>
</tr>
<tr>
<td>zetta</td>
<td>Z</td>
<td>$10^{21}$</td>
<td>$1 000 000 000 000 000 000 000 000$</td>
</tr>
<tr>
<td>exa</td>
<td>E</td>
<td>$10^{18}$</td>
<td>$1 000 000 000 000 000 000 000$</td>
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<td>peta</td>
<td>P</td>
<td>$10^{15}$</td>
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<tr>
<td>tera</td>
<td>T</td>
<td>$10^{12}$</td>
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<td>giga</td>
<td>G</td>
<td>$10^{9}$</td>
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<tr>
<td>mega</td>
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<td>kilo</td>
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<td>$10^{-1}$</td>
<td>$0.1$</td>
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<td>m</td>
<td>$10^{-3}$</td>
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<tr>
<td>micro</td>
<td>u</td>
<td>$10^{-6}$</td>
<td>$0.000 001$</td>
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<tr>
<td>nano</td>
<td>n</td>
<td>$10^{-9}$</td>
<td>$0.000 000 001$</td>
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<td>pico</td>
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<td>femto</td>
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<td>yocto</td>
<td>y</td>
<td>$10^{-24}$</td>
<td>$0.000 000 000 000 000 000 000 000 000 001$</td>
</tr>
<tr>
<td>ENGLISH UNITS</td>
<td>ENGLISH EQUIVALENTS</td>
<td>METRIC EQUIVALENTS</td>
<td></td>
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<tr>
<td>------------------</td>
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<td></td>
</tr>
<tr>
<td>1 INCH</td>
<td>= 12 INCHES</td>
<td>= about 2-1/2 CENTIMETERS</td>
<td></td>
</tr>
<tr>
<td>1 FOOT</td>
<td>= 12 INCHES</td>
<td>= about 30 CENTIMETERS</td>
<td></td>
</tr>
<tr>
<td>1 YARD</td>
<td>= 3 FEET = 36 INCHES</td>
<td>= about 1 METER</td>
<td></td>
</tr>
<tr>
<td>1 HAND</td>
<td>= about 4 INCHES</td>
<td>= about 10 CENTIMETERS</td>
<td></td>
</tr>
<tr>
<td>1 CUBIT</td>
<td>= about 1/2 YARD</td>
<td>= about 46 CENTIMETERS</td>
<td></td>
</tr>
<tr>
<td>1 BRACCIO</td>
<td>= 15 to 39 INCHES</td>
<td>= about 1/2 to 1 METER</td>
<td></td>
</tr>
<tr>
<td>1 FATHOM</td>
<td>= 6 FEET</td>
<td>= about 2 METERS</td>
<td></td>
</tr>
<tr>
<td>1 MILE</td>
<td>= 5,280 FEET</td>
<td>= about 1-1/2 KILOMETERS</td>
<td></td>
</tr>
<tr>
<td>1 OUNCE</td>
<td></td>
<td>= about 28 GRAMS</td>
<td></td>
</tr>
<tr>
<td>1 POUND</td>
<td>= 16 OUNCES</td>
<td>= about 1/2 KILOGRAM</td>
<td></td>
</tr>
<tr>
<td>1 TEASPOON</td>
<td></td>
<td>= about 5 MILLILITERS</td>
<td></td>
</tr>
<tr>
<td>1 TABLESPOON</td>
<td>= 3 TEASPOONS</td>
<td>= about 15 MILLILITERS</td>
<td></td>
</tr>
<tr>
<td>1 CUP</td>
<td>= 16 TABLESPOONS</td>
<td>= about 250 MILLILITERS</td>
<td></td>
</tr>
<tr>
<td>1 QUART</td>
<td>= 4 CUPS</td>
<td>= about 1 LITER</td>
<td></td>
</tr>
<tr>
<td>1 GALLON</td>
<td>= 4 QUARTS</td>
<td>= about 4 LITERS</td>
<td></td>
</tr>
</tbody>
</table>
Almost all developed and developing countries have a **National Metrology Institute (NMI)**, having relations with **BIPM**.

The main objectives of NMIs are to **build and maintain national standards for all measurements** carried out within that country and to **calibrate the measurement standards and devices** of lower level laboratories.

This chain extends to production, quality control and to all scientific, commercial and military devices which are used for measurements.

This way, all measurements are traceable to the national standards. National standards are linked to the standards in other countries or those of BIPM by a process of international comparisons. This is called **traceability**.

*List of NMIs in the world can be found at:* www.ume.tubitak.gov.tr/menu_links.php?f=301
Useful Links

> Bureau International des Poids et Measures (BIPM):
  www.bipm.org

> International Organization for Standardization (ISO):
  www.iso.org

> European Association of National Metrology Institutes (EURAMET):
  www.euramet.org

> Online unit converter:
  www.digitaldutch.com/unitconverter

> Scale of universe:
  http://scaleofuniverse.com
CHAPTER (3)
Analysis of Errors and Uncertainties in Measurement
## Some Terminology

<table>
<thead>
<tr>
<th>Calibration (Kalibrasyon)</th>
<th>Accuracy (Doğruluk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; Adjustment or setting of an instrument to obtain accurate readings within a reference standard</td>
<td>&gt; Degree of agreement of the measured dimension with its true magnitude</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Readability (Okunabilirlik)</th>
<th>Sensitivity (Hassasiyet)</th>
</tr>
</thead>
</table>
| > Susceptibility of an instrument for having its indications converted to a meaningful number  
\(e.g.\) converting voltage signals to electric current | > Smallest difference in a dimension that an instrument can distinguish or detect  
\(e.g.\) a scale with sensitivity of \(\pm 1\) mg |

<table>
<thead>
<tr>
<th>Precision (Kesinlik)</th>
<th>Resolution (Çözünürlük)</th>
</tr>
</thead>
</table>
| > Degree of refinement to which a measurement can be stated \(e.g.\) number of digits | > Smallest dimension/feature on a device  
\(e.g.\) resolution of LCD TV |
| > Degree of agreement within the measurements of the same quantity | > Smallest programmable step that machine can make during point to point motion |

<table>
<thead>
<tr>
<th>Repeatability (Tekrar edilebilirlik)</th>
<th>Reproducibility (Tekrar üretilebilirlik)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; Error between a number of successive attempts to move a machine to the same position</td>
<td>&gt; Degree of agreement within the individual results using the same method, the same test substance, but under different set of laboratory conditions</td>
</tr>
<tr>
<td>&gt; Ability to do the same thing over and over</td>
<td></td>
</tr>
</tbody>
</table>
Understanding the terminology

Nominal value: 10
Readings: 10.220, 9.886, 10.147, 10.126, 9.914
Average: 10.05860
Displayed: 10.06

Resolution: 0.001 units
Precision: 2 digits
Accuracy: 0.586%

Resolution

Target point

Accuracy

Positions achieved while attempting to position to target point

Accuracy vs Precision

A process or measurement can be accurate without being precise, or vice versa.

The aim is to obtain high accuracy with high precision.
Whenever a physical parameter is measured by any means, an estimate is made on the value of the quantity being measured. There are two features of such estimation: **measurement error** & **measurement uncertainty**

### Measurement Error

- It is the difference between the value of a measured quantity and a measurement estimate of its value (i.e. the accuracy of the measurement).
- This error may be systematic or random depending upon the type of measurement, the measurement device, the person making the measurement, and the environmental factors.

### Measurement Uncertainty

- Measurement errors are never known exactly. In some instances, they may be estimated and tolerated or corrected, or they may be simply acknowledged as being present.
- Whether an error is estimated or acknowledged, its existence introduces a certain amount of measurement uncertainty.
- Thus, it is a lack of knowledge concerning the error in the measurement.
Systematic Errors

> Come from the measuring instruments.

> Something is wrong with the instrument or its data handling system, or instrument is wrongly used by the experimenter.

> *errors in temperature measurements due to poor thermal contact between thermometer and substance.*

> *errors in measurements of solar radiation as trees or buildings shade the radiometer.*

A thermometer that **always reads 3 °C colder** than the actual temperature

Random Errors

> Caused by unknown and/or unpredictable changes in the experiment.

> Occur in the measuring instruments or in the environmental conditions (e.g. humidity, temperature, etc.)

> *errors in voltage measurements because of an electronic noise in the circuit of electrical instrument.*

> *irregular changes in the heat loss rate from a solar collector due to the wind.*

A thermometer that gives **random values within 3 °C** either side of the actual temperature
Evaluation of Errors

Systematic Errors

> Reproducible between measurements.

> In principle, they can be eliminated partially or completely.

> Accuracy is often reduced by systematic errors, which are difficult to detect even for experienced researchers.

They can be estimated so that the measured value can be adjusted to allow for them.

Random Errors

> Not reproducible, but fluctuate in magnitude and sign between measurements.

> We can only know the probable range over which a random error lies.

> Precision is limited by random errors, that are usually determined by repeating the measurements.

We must define their size to estimate what confidence we have in our measured value.

Standard Deviation & Least Square Method
Standard Deviation (Statistical Uncertainty)

Standard Deviation

> A measure of the spread of a probability distribution, a random variable, or multiset of values.
> More formally, it is the root mean square deviation of values from their arithmetic mean.

Sample standard deviation (\(\sigma\)) of a random variable (\(x\)) for a number of samples (\(n\)) is defined as:

\[
\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}
\]

where \(\bar{x}\) is the arithmetic mean:

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]

In practice, it is often assumed that the data are from an approximately normally distributed population. According to this, the confidence intervals are:

<table>
<thead>
<tr>
<th>(\sigma)</th>
<th>Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (\sigma)</td>
<td>68.26894921371 %</td>
</tr>
<tr>
<td>2 (\sigma)</td>
<td>95.44997361036 %</td>
</tr>
<tr>
<td>3 (\sigma)</td>
<td>99.73002039367 %</td>
</tr>
<tr>
<td>4 (\sigma)</td>
<td>99.99366575163 %</td>
</tr>
<tr>
<td>5 (\sigma)</td>
<td>99.9994266969 %</td>
</tr>
<tr>
<td>6 (\sigma)</td>
<td>99.99999980268 %</td>
</tr>
</tbody>
</table>

In order to improve the precision, sampling distribution is used:

\(\sigma \rightarrow \frac{\sigma}{\sqrt{n}}\)
Least Square Method (LSM)

Least Square Method (also called “Regression Model”)

> This is a statistical approach to estimate an expected value or function with the highest probability from the observations with random errors.

> It assumes that the **best-fit curve** is the curve that has the minimal sum of the deviations squared (least square error) from a given set of data.

> Thus, it provides the best-curve fitting (**a curve with a minimal deviation from all data points**)

Suppose that our data points are \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\) where \(x\) and \(y\) are independent and dependent values.

The fitting curve \(f(x)\) has the deviation of \(d\) from each data point:

\[
d_i = y_i - f(x_i)
\]

Hence, according to LSM, the best-fitting curve has the property that:

\[
\sum_{i=1}^{n} d_i = \text{a minimum}
\]
# Regression Methods (Trend Lines)

<table>
<thead>
<tr>
<th>Regression</th>
<th>Formulae</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>( y = mx + b )</td>
<td>A straight line is used with simple linear data sets (i.e. increasing or decreasing at a steady rate).</td>
</tr>
<tr>
<td>Logarithmic</td>
<td>( y = c \ln x + b )</td>
<td>A curved line is used when the rate of change in the data increases/decreases quickly and then levels out. Negative and/or positive values can be used.</td>
</tr>
<tr>
<td>Polynomial</td>
<td>( y = b + c_1 x + \ldots + c_n x^n )</td>
<td>A curved line is used when data fluctuates. The order is defined by number of fluctuations in the data, or by how many bends appear in the curve (e.g. Order 2 generally has one hill or valley).</td>
</tr>
<tr>
<td>Power</td>
<td>( y = cx^b )</td>
<td>A curved line is used with data sets that compare measurements increasing at a specific rate (cannot be used for data containing zero or negative values).</td>
</tr>
<tr>
<td>Exponential</td>
<td>( y = ce^{bx} )</td>
<td>A curved line is used when data values rise or fall at increasingly higher rates (cannot be used for data containing zero or negative values).</td>
</tr>
</tbody>
</table>
### How to choose the appropriate regression?

> Depends on the type of data and **the reliability of trendline**.
> It is determined by **R-squared value** (aka “the coefficient of determination”). It is an indicator from 0 to 1 that reveals how closely the estimated values correspond to the actual data.
> A trend line is **the most reliable when its R-squared value is at or near 1**.

<table>
<thead>
<tr>
<th>Regression Type</th>
<th>Equation</th>
<th>$R^2$ Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Linear</strong></td>
<td>$y = 0.119x + 1.962$</td>
<td>0.6274</td>
</tr>
<tr>
<td><strong>Logarithmic</strong></td>
<td>$y = 5.3968\ln(x) - 11.96$</td>
<td>0.7538</td>
</tr>
<tr>
<td><strong>Power</strong></td>
<td>$y = 0.0948x^{1.1108}$</td>
<td>0.8536</td>
</tr>
<tr>
<td><strong>Exponential</strong></td>
<td>$y = 1.7801e^{0.0233x}$</td>
<td>0.6449</td>
</tr>
<tr>
<td><strong>Polynomial</strong></td>
<td>$y = -0.0027x^2 + 0.4089x - 3.5419$</td>
<td>0.797</td>
</tr>
</tbody>
</table>

---

[Diagram showing raw data points and trend lines for different regression models: Linear, Logarithmic, Power, Exponential, Polynomial.]
Errors in a Manufacturing Process

- **Product error**: variation of the parts being produced ($\sigma_p$)
- **Measurement error**: associated with the measurement device and the method ($\sigma_m$)
- **Repeatability error**: associated with measurement device itself ($\sigma_e$)
- **Reproducibility error**: associated with measurement operators, methods, etc. ($\sigma_o$)

Based on ANOVA Gage R&R: $\sigma_m = \sqrt{(\sigma_p)^2 + (\sigma_o)^2}$

1. If the product with its manufacturing tolerance ($T$) conforms to a normal distribution, then: $T = 6\sigma_p$
2. Resolution of a measurement device should be **10 times better** (smaller) than resolution of dimension to be measured.
3. The measurement error should be 30% or less of the product error (i.e. $\sigma_m / \sigma_p < 30\%$)
CHAPTER (3)
Linear and Angular Measurements
Measuring tools and instruments

Direct (contact) measurement
(e.g. micrometer or caliper)

- Graduated
  (either linear or angular graduations incorporated into measuring system of the tool)
  - Rules
  - Vernier Calipers
  - Vernier Gauges
  - Micrometers
  - Protractors
  - Dial Indicators

- Non-graduated
  (gauges or adjustable tools which compare the measurements)
  - Calipers
  - Gauges and Gauge Blocks
  - Sine Bar
  - Special-purpose tools

Indirect (non-contact) measurement
(advanced methods such as optical, ultrasonic, laser, etc.)

NEXT WEEK!
Graduated Linear Measurement - Rules

Imperial steel rule with various lengths having graduations on each side

Same rule with relatively larger graduations

Metric steel rule with various lengths having graduations on each side

How to read a rule:
- A = 12 mm (12th graduation)
- B = 22 mm (22nd graduation)
- C = 31.5 mm (between 31st and 32nd)
- D = 40.5 mm (between 40th and 41st)
Graduated Linear Measurement - Vernier Calipers

How to read a vernier caliper:

- First, read the graduation on the main scale just before the vernier scale starts (i.e. 19th graduation, which gives 19 mm)
- Next, read the graduation on the vernier scale where two graduation lines on main and vernier scales perfectly match (i.e. 32nd graduation, which gives 32 * 1/50 = 0.64 mm)
- Then, add the fine reading into the main reading (i.e. 19 + 0.64 = 19.64 mm)

More precise tools capable of measuring external and internal dimensions as well as depths.
Graduated Linear Measurement - Vernier Caliper

Direct reading of an internal length using digital vernier caliper

Direct reading of an external length using digital vernier caliper

Vernier caliper with a dial indicator
Graduated Linear Measurement - Vernier Height Gauges

Designed for use in toolrooms, workshops, inspection departments to measure or mark off vertical heights and locating center distances.
Graduated Linear Measurement - Vernier Depth Gauges

Designed for use in toolrooms, workshops, inspection departments to measure depths of holes, slots, recesses, and so on.
Graduated Linear Measurement - Outside Micrometers

How to read a vernier metric micrometer:

- **Sleeve div.** = 1 mm
- **Thimble div.** = 1/50th of sleeve sub-div. = 1/100 mm
- **Vernier div.** = 1/10th of thimble div. = 1/1000 mm

A. The highest figure: 5 * (sleeve div.) = 5 mm
B. The half-figures: 1 * (sleeve sub-div.) = 0.5 mm
C. The highest figure: 28 * (thimble div.) = 0.28 mm
D. The matching figure: 3 * (vernier div.) = 0.003 mm

**FINAL READING** = A + B + C + D = 5.783 mm
Graduated Linear Measurement - Outside Micrometers

Dial-indicating Micrometer

V-anvil Micrometer (measuring odd-fluted taps, milling cutters, reamers, and checking out of roundness)

Direct-reading Micrometer

Screw Thread Micrometer (measuring pitch diameter of screw threads)
Graduated Linear Measurement - *Inside Micrometers*

Standard Inside Micrometers

Digital Inside Micrometers

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Graduated Linear Measurement - *Micrometer Bore and Depth Gauges*

**Inside Micrometer Set**

**Micrometer Hole and Bore Gauges**

**Micrometer Depth Gauges**
Graduated Angular Measurement - Protractors

Simple Protractor (measuring angles from 0 to 180°)

Universal Bevel Protractor (main scale consists of 4 portions of 90°)

How to read an angle on a bevel protractor:
- Main div. = 1° = 60′
- Vernier div. = 1/12th of main div. ≈ 0.0833° = 5′

- The highest figure: 50° (main div.)
- The matching figure: 4° (vernier div.) ≈ 0.333° = 20′
- The final reading is: ≈ 50.333° or 50° 20′

Measuring acute (a) and obtuse (b) angles
Special Purpose Measurement Tools

**Level** *(used for setting up and testing machinery)*

**Combination Set** *(consisting of a 90° square, a level, a depth/height gauge and a bevel protractor with a centring head)*

**Dial test indicator on a magnetic stand** *(used as an accessory with machine tools for accurate and delicate measurements)*
Non-Graduated Linear Measurement - Calipers

- Standard calipers have a fine adjustment screw and a quick-adjusting spring nut.
- Accuracy obtained with these tools depends mostly on the inherent skill of users.
- The measurements are carefully transferred to a graduated measuring tool.

Caliper for inside measurement

Caliper for outside measurement

Caliper used as a divider
Non-Graduated Linear Measurement - Gauges

Telescopic Gauges (consisting of a handle with two plunger-contacts at right angles which both lock simultaneously)

Small Hole Gauges (consisting of an expanding ball head adjusted to size by a knurled knob to provide an accurate “feel” for obtaining measurements in a hole or slot)

Surface Gauge (consisting of a ground rectangular base with a round upright rod and a fine adjustment feature in the base. It can be used in a layout work for marking lines on vertical or horizontal surfaces, and also used in inspection work as height or depth gauge)
Non-Graduated Linear Measurement - Special Purpose Gauge

Screw Pitch Gauges (consisting of a metal case containing many separate leaves. Each leaf has teeth corresponding to a definite pitch. By matching the teeth with the thread on work, the correct pitch can be read directly from the leaf)

Tap and Drill Gauges (consisting of a flat rectangular steel plate with holes accurately drilled and identified according to their size)

Radius Gauges (available as individual leaves and each leaf is marked with its radius. They are designed to check both convex and concave radii)
Non-Graduated Linear Measurement - *Rectangular Gauge Blocks*

Linear Gauge Blocks: (a) rectangular, (b) square, (c) square with center hole
Non-Graduated Angular Measurement - Angle Gauge Blocks

Complete set of angle gauge blocks

Setting a revolving magnetic chuck using angle gauge blocks

True Square
Non-Graduated Angular Measurement - Sine Bar

How to use Sine Bar:

- We want to set an angle of 14° 12' using a 10 mm sine bar.
- This means: \( l = 10 \text{ mm} \) and \( \theta = 14.2^\circ \)
- So: \( h = l \cdot \sin \theta = 2.453 \text{ mm} \)
- Thus, a combination of gauge blocks providing this height must be used.

Limitations of Sine Bar:

- When using a sine bar, the height setting is limited by available gauge block divisions. This causes an error that may be negligible or quite significant depending on the accuracy of measurement.
- Due to the nature of trigonometry; at larger angles, the sine bar is susceptible to errors in the length of sine bar as well as in the height of gauge blocks.
Engineering Metrology

CHAPTER (4)
Statistical Quality Control and Use of Control Charts
## Statistical Process Control (SPC) & Control Charts

### Definition of SPC

> SPC uses statistical tools to **observe the performance of production process** in order to predict significant deviations that may later result in rejected product.

> To control a process based on varying data, it is necessary to keep a check on the current state of **accuracy (central tendency)** and **precision (spread)** of the distribution of data.

### Achievement of SPC

> SPC is achieved with the aid of **control charts** *(pioneered by Walter A. Shewhart in 1920)*.

> The most frequently used charts are **Mean and Range Charts**, which are used together.

### Implementation of Control Charts

> The operation of control charts to detect the state of control of a process is as follows:

  - Periodically, **samples of a given size** *(four steel rods, five tins of paint, eight tablets, four delivery times, etc.)* are taken from the process at reasonable intervals, when it is believed to be stable or in-control and adjustments are not being made.

  - **The variable** *(length, volume, weight, time, etc.)* is measured for each item of the sample so that the sample mean and range values are recorded on a chart.

  - Control limits are determined in order to check stability of the process.

  - If the process is not in control, efforts are made for making the process under control.

  - The process capability for specified tolerances is examined using capability indices.
A Case Study

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Rod Lengths (mm)</th>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>iv</th>
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<tbody>
<tr>
<td>1</td>
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</tbody>
</table>

Suppose that, for production of rods, we have **100 rod lengths** as **25 samples** of size 4 were taken.

Let's represent this data using a **histogram chart**, which is used for showing the frequency of data values. First, we must define **the bins** (i.e. intervals).

For this purpose, we search for **the min. and the max.** values in the data set (which are **134** and **160**).

Then, we define starting & end values for the bins (i.e. **133.5** & **160.5**). After this, we divide the difference between these values by equal intervals (i.e. 27/3 which gives **9 equal intervals**). Therefore, the bins for this data set can be:

- 133.5 – 136.5
- 136.5 – 139.5
- 139.5 – 142.5
- 142.5 – 145.5
- 145.5 – 148.5
- 148.5 – 151.5
- 151.5 – 154.5
- 154.5 – 157.5
- 157.5 – 160.5

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The histogram chart is obtained using **Microsoft Excel** software *(Click on the “Data Analysis” under “Tools” menu and choose “Histogram” option from the list. If you can’t see “Data Analysis” function, then go to “Add-ins” under “Tools” menu and click on “Analysis Toolpak”)*.

Select the data set (i.e. 100 rod lengths) and the intervals so that the histogram chart will be built. The frequency of each value within the data set according to each bin will be calculated automatically.

Sometimes, it is more practical to show midpoints in the chart rather than intervals. In such cases, the midpoint of each bin is calculated and used in the chart (e.g. mean of interval 133.5 – 136.5 is 135).

Histogram charts are also useful for showing the distribution of data sets.
### Sample Mean & Grand (Process) Mean

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Rod Lengths (mm)</th>
<th>Sample Mean (mm)</th>
</tr>
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<tbody>
<tr>
<td>i</td>
<td>ii</td>
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<td>140</td>
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</table>

For the given data, it is not meaningful to calculate the mean of a whole population (i.e. the total output from a process rather than a sample):

\[ \mu = 150.1 \text{mm} \]

Instead, “Grand (Process) Mean” (i.e. the mean of all sample means) is a good estimate of population mean:

\[ \bar{X} = \frac{\sum_{i=1}^{k} X_i}{k} \]

First, the mean of each sample is calculated, e.g. mean of sample no. 9 is:

\[ \bar{X} = \left( \frac{158 + 150 + 149 + 156}{4} \right) = 153.25 \text{mm} \]

Then, the process mean is calculated as:

\[ \bar{X} = \left( \frac{147.50 + 147.00 + 144.75 + ... + 150.50}{25} \right) = 150.1 \text{mm} \]
Standard Error (Deviation) of Means

Based on the information in previous slides, it is now better to use “the standard deviation of sample means” instead of using “standard deviation of the population”.

Standard Error (SE) of Means is defined as:

\[ SE = \frac{\sigma}{\sqrt{n}} \]

where \( \sigma \) is the standard deviation of population, \( n \) is the sample size (total number of samples). In case of our example, \( n \) is 4.

SE of means is more reliable than SD of whole population. As seen from the graph, the scatter (i.e. the precision) of the sample means is much less than the scatter of the individual rod lengths. On the other hand, the mean (i.e. the accuracy) remains unchanged as population is the same:

\[ \mu = \bar{X} = 150.1 \text{ mm} \]
It is possible to simplify the calculation of warning and action limits. In statistical process control for variables, the sample size is usually less than 10; so it is possible to use an alternative measure of spread of process: the mean range of samples.

The range (i.e. difference between the highest and the lowest observations) is the simplest possible measure of scatter:

\[
R = \frac{1}{k} \sum_{i=1}^{k} R_i
\]

For instance, the range in sample no. 9 is the difference between the longest (158 mm) and the shortest (149 mm), which is 9 mm. Hence, the mean range (i.e. the mean of all sample ranges) is found as:

\[
R = \frac{(10 + 19 + 13 + \ldots + 17)}{25} = 10.8\text{mm}
\]
Building X-Bar (Mean) Chart

First of all, we draw the distribution of sample means graph, and then turn this bell-shape graph onto its side having process mean in x-axis and sample mean in y-axis.

In a stable process, most of the sample means lie within the range of $\bar{X} \pm 3SE$. So, we extrapolate the process mean by $\pm 3SE$ on both sides in order to obtain “upper and lower action limits”. If the process is running satisfactorily, we expect from the normal distribution that at least 0.27% ($\approx 1$ in 370) of the means of successive samples should lie out of action limits.

We can extrapolate process mean by $\pm 2SE$ to obtain “upper and lower warning limits”. Thus, at least 4.45% ($\approx 1$ in 22) of the means of successive samples should lie out of warning limits.

Chance of having two consecutive sample means in the warning zone is about $1/22 \times 1/22 = 1/484$, which is even lower than the chance of one value in action zone, suggests that the process is out of control.
Calculation of Action and Warning Limits

Standard deviation can be expressed in terms of the mean range and Hartley’s constant (d_n or d_2):

\[ \sigma = \frac{R}{d_n} = \frac{R}{d_2} \]

Hence, when the limits are being calculated, the mean range can be used instead of SE:

**Action Limits**

\[ \bar{X} \pm 3SE = \bar{X} \pm 3 \frac{\sigma}{\sqrt{n}} = \bar{X} \pm \frac{3}{d_n \sqrt{n}} R \]

**Warning Limits**

\[ \bar{X} \pm 2SE = \bar{X} \pm 2 \frac{\sigma}{\sqrt{n}} = \bar{X} \pm \frac{2}{d_n \sqrt{n}} R \]

As d_n and n are all constants for the same sample size, it is possible to replace numbers and symbols with just one constant, called A_2:

\[ \frac{3}{d_n \sqrt{n}} = A_2 \quad \text{and} \quad \frac{2}{d_n \sqrt{n}} = 2/3 A_2 \]

As a result, action and warning limits can be written in terms of the mean range and a constant:

**Warning Limits** = \( \bar{X} \pm 2/3 A_2 R \)  &  **Action Limits** = \( \bar{X} \pm A_2 R \)
Constants for Mean Charts

The constants (\(d_n\) and \(A_2\)) for sample sizes (\(n\)) from 2 to 12 are available.

For the sample sizes up to 12, the range method of estimating action and warning limit is relatively efficient.

For the values greater than 12, the range loses its efficiency rapidly as it ignores all the information in the sample between the highest and the lowest values.

The sample sizes of 4 or 5 are generally employed in control charts, which gives satisfactory results.

<table>
<thead>
<tr>
<th>Sample Size (n)</th>
<th>Hartley’s Constant ((d_n \text{ or } d_2))</th>
<th>Constant for use with Sample Range</th>
<th>(A_2)</th>
<th>(2/3 A_2)</th>
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</table>

In case of our example, the sample size (\(n\)) is 4. Therefore, the constants (\(d_n\) and \(A_2\)) are selected accordingly.
Case Study – Mean Chart

The mean chart is constructed by turning the bell-shape histogram chart onto its side, hence we have **sample mean as y-axis** and **the process mean as x-axis** (which was calculated before as 150.1 mm).

Then, action and warning limits are determined based on mean range and constants:

- Mean Range (calculated): $R = 10.8\ mm$
- Hartley's Constant (from table): $d_n = 2.059$
- Sample Range Constants (from table): $A_2 = 0.73$ & $2/3A_2 = 0.49$

**Upper Warning Limit:** $\bar{X} + 2/3A_2 R = 155.40\ mm$
**Lower Warning Limit:** $\bar{X} - 2/3A_2 R = 144.81\ mm$

**Upper Action Limit:** $\bar{X} + A_2 R = 157.98\ mm$
**Lower Action Limit:** $\bar{X} - A_2 R = 142.22\ mm$
Building R-bar (Range) Chart

The control limits on a range chart are asymmetrical about the mean range since the distribution of sample ranges is a positively skewed distribution.

Thus, upper and lower action limits are calculated for 0.1% & 99.9% of sample range while upper and lower warning limits lie at 2.5% & 97.5%:

- Upper Action Limit: \( D^{'\text{0.001}} \cdot R \)
- Lower Action Limit: \( D^{'\text{0.999}} \cdot R \)
- Upper Warning Limit: \( D^{'\text{0.025}} \cdot R \)
- Lower Warning Limit: \( D^{'\text{0.975}} \cdot R \)

The constants for calculating control limits are available for sample sizes \( (n) \) up to 12.

For our case, the constants are selected from the table for sample size of 4.

<table>
<thead>
<tr>
<th>Sample Size (n)</th>
<th>Constant for use with Mean Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( D^{'\text{0.999}} )</td>
</tr>
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<td>0.35</td>
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<td>11</td>
<td>0.38</td>
</tr>
<tr>
<td>12</td>
<td>0.40</td>
</tr>
</tbody>
</table>
Case Study – Range Chart

The range chart is constructed by turning the bell-shape histogram chart onto its side, hence we have sample range as y-axis and mean range as x-axis (which was calculated before as 10.8 mm).

Then, action and warning limits are determined based on mean range and constants:

\[
\begin{align*}
D_{0.001}' &= 2.57 \\
D_{0.999}' &= 0.10 \\
D_{0.025}' &= 1.93 \\
D_{0.975}' &= 0.29
\end{align*}
\]

from table

Upper Warning Limit: \(D_{0.025}' R = 20.8 \text{mm}\)

Lower Warning Limit: \(D_{0.975}' R = 3.1 \text{mm}\)

Upper Action Limit: \(D_{0.001}' R = 27.8 \text{mm}\)

Lower Action Limit: \(D_{0.999}' R = 1.1 \text{mm}\)
Is the process in control?

✓ NO mean or range values which lie outside the action limits (action zone)?
✓ NO more than about 1 in 22 values between warning and action limits (warning zone)?
✓ NO incidence of two consecutive mean or range values which lie outside the same warning limit on either mean or range chart (warning zone)?
✓ NO run or trend of five or more consecutive mean or range values, which also infringes a warning or action limit (warning zone and/or action zone)?
✓ NO run of more than six sample means which lie either above or below the process mean (stable zone)?
✓ NO trend of more than six values of the sample means which are either rising or falling (stable zone)?

PROCESS IS IN CONTROL !!!

MEAN CHART

RANGE CHART
(should be examined first)
What if the process is out of control?

Do not use the control charts and investigate the assignable causes of variation.

The assignable causes of variation have been identified, and now to be eliminated.

Another set of samples from the process is taken and control limits are recalculated.

Approximate control limits are recalculated by simply excluding the out of control results for which special causes have been found and corrected.

The exclusion of samples representing unstable conditions is not just throwing away bad data. By excluding the data affected by known causes, we have a better estimate of variation due to common causes only.

The process is re-examined to see if it is in statistical control.

If the process is shown to be in statistical control, the next task is to compare the limits of this control with the required tolerance.
Process Variability and Tolerances

Control Limits vs. Tolerance Limits

> Tolerance limits should be based on the functional requirements of the product. Limits on control charts are based on the stability and actual capability of the process.

> A process may not meet the specification requirements, but still be in a state of statistical control.

> A comparison of process capability and tolerance can only take place, with confidence, when the process is statistically in control. Thus, in controlling a process, it is necessary to establish first that it is in statistical control and then to compare its centring & spread with the specified target value and the specification tolerance.

The relationship between process variability and tolerances can be formalized by consideration of standard deviation of the process. In order to manufacture within the specification, distance between Upper Specification Limit (USL) or Upper Tolerance (+T) and Lower Specification Limit (LSL) or Lower Tolerance (–T) must be analyzed.

There are three precision levels:

> High Relative Precision: 2T >> 6σ
> Medium Relative Precision: 2T ≥ 6σ
> Low Relative Precision: 2T < 6σ
## Process Capability Indices

### Process Capability Index

- It is a **measure relating the actual performance of a process to its specified performance**, where the processes are considered to be a combination of plant or equipment, the method itself, the people, the materials, and the environment.

- Process capability indices are simply a means of indicating **the variability of a process relative to the product specification tolerance**.

Calculation of process capability indices **estimates the short-term variations** within the process. This short-term is the period over which the process remains relatively stable. However, we know that processes do not remain stable for all time, and therefore we need to allow within the specified tolerance limits for:

- some movement of the mean
- detection of changes of the mean
- possible changes in the scatter (range)
- detection of changes in the scatter
- possible complications of non-normal distributions
Relative Precision Index (RPI)

This is the oldest index based on a ratio of the mean range of samples to the tolerance band. In order to avoid the production of defective material, the specification width must be greater than the process variation: \[ 2T \geq 6\sigma \]

We know that: \[ \sigma = \frac{\bar{R}}{d_n} = \text{Mean of Sample Ranges} \quad \text{Hartley's Constant} \]

Then, we obtain: \[ \frac{2T}{\bar{R}} \geq \frac{6}{d_n} \]

\(2T / \bar{R}\) is known as Relative Precision Index (RPI) and the value of \(6 / d_n\) is the minimum RPI to avoid the production of a material outside the specification limits. Moreover, RPI index does not comment on the centring of a process as it deals only with its relative spread or variation.

In our case study: Min. RPI = \[ \frac{6}{d_n} = \frac{6}{2.059} = 2.914 \]

If we are asked to produce rods within ±10 mm of the target length:

\[ \Rightarrow 2T = 20 \text{ mm} \quad \Rightarrow \quad \text{RPI} = \frac{2T}{\bar{R}} = \frac{20}{10.8} = 1.852 \quad \Rightarrow \quad \text{Reject Material} \]

If the specified tolerances are widened to ±20 mm:

\[ \Rightarrow 2T = 40 \text{ mm} \quad \Rightarrow \quad \text{RPI} = \frac{2T}{\bar{R}} = \frac{40}{10.8} = 3.704 \quad \Rightarrow \quad \text{Avoid Rejecting Material} \]
Cₚ Index

In order to manufacture within a specification, difference between USL and LSL must be less than total process variation. So, a comparison of 6σ with (USL - LSL) or 2T gives Cₚ index:

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{2T}{6\sigma}$$

(a) $C_p < 1 \quad \Rightarrow \quad$ Process variation is greater than tolerance band, so the process is incapable.

(b) $C_p \geq 1 \quad \Rightarrow \quad$ For increasing values of Cₚ, the process becomes increasingly capable.

Similar with RPI, Cₚ index makes no comment about the centring of the process, which means that it is a simple comparison of total variation with tolerances.
**Cpk Index**

A situation like this invalidate the use of Cp index, hence there is a need for another index which takes account of **both the process variation and the centring**. Such an index is called **Cpk index**, which is widely accepted as a means of communicating process capability.

For USL and LSL limits, there are two Cpk values (Cpk_u and Cpk_l). These relate the difference between process mean and USL/LSL respectively, and the lesser of these two will be the value of Cpk index:

\[
C_{pk} = \text{lesser of } \left( C_{pk_u} = \frac{USL - \bar{X}}{3\sigma} \right) \text{ or } \left( C_{pk_l} = \frac{\bar{X} - LSL}{3\sigma} \right)
\]

- **Cpk < 1** means that, considering the process variation and its centring, at least one of the tolerance limits is exceeded and **the process is incapable**.
- As in the case of Cp, **increasing values of Cpk correspond to increasing capability**.
- It may be possible to **increase Cpk value by centring the process** so that its mean value coincides with the mid-specification (target).
- Comparison of Cp and Cpk shows **no difference if the process is centred** on the target.
- **Cpk can be used when there is only one specification limit**, but Cp cannot be used in such cases.
Engineering Metrology

CH (6)
Measurement of Geometric Shape
Measurement of Roundness using Intrinsic Datum

The conventional method using the points on the surface of the part for a reference is called “intrinsic” datum system.

For large shafts

For large bores
Measurement of Roundness using Extrinsic Datum

In such system, measurements are taken based on an external reference of known precision.
Analysis of Roundness using Reference Circles

There are four types of reference circle in the analysis of Peak to Valley out-of-roundness (RON):

> **Least Squares Reference Circle (LSCI):** A circle is fitted such that the sum of squares of the deviations is a minimum. RON is the distance from the highest peak to the lowest valley.

> **Minimum Zone Reference Circles (MZCI):** Two concentric circles positioned to enclose the measured profile such that their radial deviation is a minimum. RON is the radial separation of the two circles.

> **Minimum Circumscribed Circle (MCCI):** The circle of minimum radius enclosing the profile (aka Ring Gauge Reference Circle). RON is the maximum deviation of the profile from this circle.

> **Maximum Inscribed Circle (MICI):** The circle of maximum radius enclosing the profile (aka Plug Gauge Reference Circle). RON is the maximum deviation of the profile from this circle.
Defining Least Squares Circle

> Equally-spaced radial ordinates are drawn relative to polar roundness diagram.

> The rectangular coordinates of intersection of each point \((X & Y)\) are measured.

> Then, the updated coordinates of each point \((X' & Y')\) are calculated based on center of least square circle \((a & b)\).

> After that, radial distance \((r)\) for each point is obtained.

> Finally, the radius of LSC \((R)\) is defined.

\[
\begin{array}{c|ccc|c}
\text{No.} & X & Y & X' & Y' & r \\
\hline
1 & 0.00 & 1.77 & 0.090 & 1.675 & 1.677 \\
2 & 0.83 & 1.43 & 0.740 & 1.335 & 1.526 \\
3 & 1.28 & 0.71 & 1.190 & 0.615 & 1.340 \\
4 & 1.50 & 0.00 & 1.410 & 0.095 & 1.413 \\
5 & 1.45 & -0.84 & 1.360 & -0.935 & 1.650 \\
6 & 0.73 & -1.31 & 0.640 & -1.405 & 1.544 \\
7 & 0.00 & -1.25 & 0.090 & -1.345 & 1.348 \\
8 & -0.65 & -1.10 & -0.740 & -1.195 & 1.406 \\
9 & -1.29 & -0.74 & -1.380 & -0.835 & 1.613 \\
10 & -1.42 & 0.00 & -1.510 & 0.095 & 1.513 \\
11 & -1.14 & 0.63 & -1.230 & 0.535 & 1.341 \\
12 & -0.75 & 1.27 & -0.840 & 1.175 & 1.444 \\
\end{array}
\]

\[
a = \frac{2\Sigma x}{n} = 0.090 \quad b = \frac{2\Sigma y}{n} = 0.095
\]

\[
r = \sqrt{(X')^2 + (Y')^2} \quad R = \frac{\sum r}{n} = 1.485
\]
**Other Parameters**

**Eccentricity**

> The position of center of a profile relative to a datum point.
> It is a vector quantity with magnitude and direction.
> The magnitude is the distance between profile center (center of the fitted reference circle) and the datum point.
> The direction is an angle from the datum point.

**Concentricity**

> The diameter of circle described by the profile center when rotated about the datum point.
> It has only magnitude and no direction.

**Runout**

> The radial difference between two concentric circles centered on the datum point which are drawn such that one coincides with the nearest and the other coincides with the farthest point on the profile.
> It combines the effect of form error and the concentricity to give a predicted performance when rotated about a datum.
Other Parameters

**Flatness**
- The peak-to-valley departure from a reference plane.
- Either least square (LS) or minimum zone (MZ) can be used.

**Squareness**
- The minimum axial separation of two parallel planes normal to the reference axis and which totally encloses the reference plane.
- Either least square (LS) or minimum zone (MZ) can be used.

**Coaxiality**
- The diameter of a cylinder that is coaxial with the datum axis and just encloses the axis of the cylinder referred for coaxiality evaluation.

**Cylindricity**
- The minimum radial separation of two cylinders, coaxial with the fitted reference axis, which totally encloses the measured data.
- Either least square, minimum zone, maximum inscribed or minimum circumscribed cylinders (LSCY, MZCY, MICY, MCCY) can be used.
# Measurement of Geometric Shapes

<table>
<thead>
<tr>
<th>Laser Interferometry</th>
<th>Autocollimator</th>
<th>Laser Alignment</th>
<th>Electronic Level and Clinometer</th>
<th>Dial Indicator Circular Tracing</th>
<th>Talyrond 365</th>
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<tr>
<td>Flatness</td>
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<td>Squareness</td>
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<td>Roundness</td>
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<td>Coaxiality</td>
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<td>Eccentricity</td>
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<td>Runout</td>
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# Rotational Measurement

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<td><img src="image6.png" alt="Diagram" /></td>
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### Rotational Measurement

<table>
<thead>
<tr>
<th>Flatness (Single Circumference)</th>
<th>Flatness (Multiple Circumference)</th>
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<thead>
<tr>
<th>Squareness (Against Axis)</th>
<th>Squareness (Against Plane)</th>
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<th>Parallelism (Single Radius)</th>
<th>Parallelism (Multiple Radius)</th>
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## Rotational Measurement

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<td><img src="" alt="Diagram" /></td>
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<td>Circular run-out</td>
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<thead>
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<td><img src="" alt="Diagram" /></td>
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<table>
<thead>
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<td>d max - d min</td>
<td>MZC</td>
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<td>d max</td>
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### Rectilinear Measurement

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<th>FLATNESS</th>
<th>SQUARENESS</th>
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<td><img src="image1.png" alt="Image" /></td>
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<tr>
<td><strong>STRAIGHTNESS (VERTICAL)</strong></td>
<td><strong>SQUARENESS (AGAINST AXIS)</strong></td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td><strong>STRAIGHTNESS (HORIZONTAL)</strong></td>
<td><strong>SQUARENESS (AGAINST PLANE)</strong></td>
</tr>
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<td><img src="image5.png" alt="Image" /></td>
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Engineering Metrology - Professor Nabil Gadallah
Rectilinear Measurement

<table>
<thead>
<tr>
<th>PARALLELISM (VERTICAL)</th>
<th>PARALLELISM (HORIZONTAL)</th>
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<tr>
<td><img src="image1" alt="Diagram" /></td>
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<table>
<thead>
<tr>
<th>TAPER RATIO (VERTICAL)</th>
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<table>
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<th>SLOPE (VERTICAL)</th>
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# Rectilinear Measurement

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<thead>
<tr>
<th>CYLINDRICITY</th>
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<td><img src="image3" alt="Cylindrical Shape" /></td>
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<td><strong>r1 - r2</strong></td>
<td><strong>r1 - r2</strong></td>
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<table>
<thead>
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<th>TOTAL RUN-OUT (RADIAL)</th>
<th>TOTAL RUN-OUT (AXIAL)</th>
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<tr>
<td><strong>Total run-out</strong></td>
<td><strong>Squareness</strong></td>
</tr>
</tbody>
</table>

*Engineering Metrology - Professor Nabil Gadallah*
CHAPTER (6)
Advanced Measurement Systems
Coordinate Measuring Machine (CMM)

Coordinate Measuring Machines (CMMs) provide precise and accurate measurements of 3D coordinates of components using small-head touching probes via movable axes.

Diagram of a Coordinate Measuring Machine with labels:
- Sleeve
- Support
- Portal
- Probing head probe
- Granite table
- Rotary table
- Damping system

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5-Axis Scanning Applications using CMM

Helix Scan

Gasket Scan

Blade Sharp Edge Sweep Scan

Rapid Head Touches

Sweep Scan

Round the Blade Scan
Use of Light

- Electromagnetic radiation (EMR) consists of self-propagating electromagnetic waves (such as light), which can travel through vacuum, and sometimes matter as well.

- EMR has electrical and magnetic field components, which oscillate in phase perpendicular to each other and to the direction of propagation of the wave.

- EMR behaves as both wave and particle. The particles are photons. As waves, EMR has both frequency and wavelength.

- Electromagnetic spectrum is the range of all electromagnetic frequencies.

- Frequencies can be grouped by their use (such as radio waves, visible light, x-rays, microwaves, etc.)

- Chart shows the range of electromagnetic spectrum and common uses for different parts of the spectrum. Frequency is shown on the left, and wavelength is on the right.

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Introduction to Interferometry

> Interferometry is the technique of superimposing (i.e. interfering) two or more waves in order to detect differences between them.

> It is applied in a wide variety of fields including astronomy, fiber optics, optical metrology, seismology, oceanography, quantum mechanics, and plasma physics.

> From metrology viewpoint, it is a series of non-contact techniques using the interference of light waves to determine surface shape and transmission properties.

Principle of Interferometry

> Whenever two waves comes together at the same time and place, interference occurs.

> If both waves are in phase (i.e. their crests coincide), they will add together to form a single wave with higher crest (i.e. larger amplitude). This is called constructive interference.

> Destructive interference occurs when the waves are out of phase (i.e. the crest of one wave coincides with the through of other wave). Therefore, the amount of interference depends on both the amplitudes of those waves and their frequencies (the degree to which their respective crests are in phase with each other).
Principle of Interferometry

- Simple interferometry setup includes the use of parallel beam of monochromatic light.

- For this purpose, **an optical flat** (a disc of stress-free glass or quartz with a highly polished surface) is placed on top of the surface to be measured at a very small angle of \( \theta \).

- The light beam from source \( S \) is projected onto the optical flat. Two reflected components of light wave (partially reflected from \( a \) and \( b \)) are collected, and hence the combined view is obtained.

- Further along the surface at a distance of half-wavelength \( (\lambda/2) \) and due to the angle \( \theta \), the ray (light beam) leaving source \( S \) will again split into two components whose path lengths are different.

- Therefore, the surface will be crossed by a pattern of dark bands (fringes), which are **straight for the case of a flat surface**.

\[
\sin \theta = (\frac{\lambda}{2}) \times t
\]
Fringe Analysis

**Straight Fringe Pattern:**
- Such pattern indicates **a very flat surface**.
- Suppose that the wavelength ($\lambda$) is 0.6328 µm.
- Thus, **the flatness** is 0.3164 µm.

**Curved Fringe Pattern:**
- In such cases, **two red lines** tangent to the center of two adjacent fringes are drawn. **The blue line** indicates the center of a single fringe.
- If the distance between red lines ($a$) is 5.02 µm and the distance between red and blue lines ($b$) is 1.24 µm, then, **the flatness** is $\approx 0.078$ µm.

**Circular Fringe Pattern:**
- This pattern is obtained when the surface to be measured is **convex** or **concave**.
- So, the total distance between the peak and the deepest point (i.e. height) can be calculated.
- Since there are **5 fringes**, then **the height** is found to be 1.582 µm.

**Flatness**
$$\text{Flatness} = \frac{\lambda}{2}$$

**Flatness**
$$\text{Flatness} = \frac{\lambda}{2} \times \left(\frac{b}{a}\right)$$

**Height**
$$\text{Height} = \frac{\lambda}{2} \times \text{(number of fringes)}$$
Fringe Analysis Software

In most cases, fringe patterns are quite sophisticated. Therefore, special-purpose software with advanced techniques may be required for accurate fringe analysis.
Common Types of Interferometers

Various systems are available for different type of measurements based on the application area.

Michelson Interferometer

Sagnac Interferometer

Mach-Zehnder Interferometer

Fabry-Perot Interferometer

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Digital (Light) Microscopy

Nikon Eclipse ME-600 Digital Microscope (1.3 megapixel color resolution imaging at a rate of 15 fps)
Electron Microscopy

Recently emerging as an advanced technology in many engineering fields, electron microscopy allows to visualize objects that are as small as 0.1 nm (1Å).

Unlike light microscopy, the object is not illuminated with light, but bombarded by electrons.

Transmission Electron Microscopy (TEM) is used to analyse the inner structure of objects (such as tissues, cells, viruses) while Scanning Electron Microscopy (SEM) is used to visualize the surface of tissues, macromolecular aggregates and materials.

Both TEM and SEM use a source (an electron gun), lenses (electro-magnetic lenses that deflect electrons), a condenser (that concentrates beam), and an objective to focus (the thing to be measured).
Electron Microscopy Examples

- Pollen
- Escherichia coli bacteria
- Nylon stocking fibres
- Micro-scale circuiting
- Uranium waste bacteria
- Witherite (barium carbonate crystal)
Use of Laser

> The term “laser” is an acronym for “Light Amplification by Stimulated Emission of Radiation”.

> It is a device that emits light (electromagnetic radiation) through a process of optical amplification based on the stimulated emission of photons.

> Different types of laser are used for research and industrial purposes (such as metal cutting, welding, dynamic alignment or positioning, dimensional measurements, nondestructive testing, lithography, medical treatments, military applications, and so on).

> For more information on type of lasers and their specific use: [http://en.wikipedia.org/wiki/Laser_types](http://en.wikipedia.org/wiki/Laser_types)
Laser Displacement Sensors with CCD Detectors

Laser-triangular sensor uses a high-speed, noncontact line-range laser probe to perform complex-part profile measurements. Laser beam is spread out into a plane that forms a line of light on the part surface. CCD (Charge Coupled Device) sensor mounted in the probe takes range measurements over the entire line of laser light at the same time (instead of measuring just one point), which dramatically decreases scan time.

ABLE
ABLE intelligently controls the three elements of laser emission time, laser power, and gain (CCD amplification factor).

LI-CCD
Demonstrates higher capacities in accuracy, speed, and sensitivity.

2-way Optical System
Two types are available: the wide-spot type with excellent measurement stability and the small-spot type suitable for minute targets and profile measurements.

High-accuracy lens unit
The high-accuracy Elnostar lenses integrated with the sensor head achieves highly accurate and highly stable measurements.
Laser Interferometers

Compared with common interferometers, laser interferometry provides precise and accurate optical measurements as well as positioning and movement systems.

Laser Interferometer for Vision

Laser Interferometer System for Precise Positioning and Accurate Measurement
Use of Sound

- Sound is a sequence of waves of pressure propagating through a compressible media (such as air, water or solids).
- During propagation, waves are reflected, refracted or attenuated by the medium.
- Sound waves are classified into specific ranges according to their frequencies: infrasound, acoustic, and ultrasound.
- **Infrasound** has the frequency of less than 20 Hz, which is the normal limit of human hearing. The sound having frequencies of greater than 20 kHz is **ultrasound**, which is not audible by humans.
- **Mach number** \( (M = \frac{V}{C}) \) is used to define speed of an object travelling in a medium \( (V) \) in relation with speed of sound \( (C) \).
- From metrology viewpoint, sounds waves with certain frequencies and amplitudes are used in various applications (such as underwater, seismology, medical, NDT, infrasound & ultrasonic inspection).

---

![Frequency vs. Sound](image)

<table>
<thead>
<tr>
<th>Regime</th>
<th>Subsonic</th>
<th>Transonic</th>
<th>Sonic</th>
<th>Supersonic</th>
<th>Hypersonic</th>
<th>High-hypersonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach</td>
<td>&lt; 1.0</td>
<td>0.8 - 1.2</td>
<td>1.0</td>
<td>1.2 - 5.0</td>
<td>5.0 - 10.0</td>
<td>&gt; 10.0</td>
</tr>
</tbody>
</table>

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SONAR

- SONAR stands for “SOund NAvigation and Ranging”.
- It is a technique using underwater sound propagation to navigate, communicate with or detect the objects.
- It directs a beam of sound waves downward. After the sound wave hits bottom of ocean or an object, it will bounce off and return back causing an echo. This is recorded on a depth recorder on the ship.
- There are two SONAR systems: Active (deploys and receivies its own signal) & Passive (just listening)
Ultrasonic Testing

> Ultrasonic Testing (UT) uses high frequency sound energy to conduct flaw detection, dimensional inspection, material characterization, and more.

> Typical UT system consists of a pulser/receiver, a transducer, and a display software/device. Pulser/receiver produces high voltage electrical pulses. Driven by the pulser, the transducer generates high frequency ultrasonic energy, which propagates through the part in form of waves. When there is discontinuity (such as crack/flaw) in the wave path, some part of energy is reflected back from the flaw surface. Based on voltage signals of reflecting energy, location and size of flaws are displayed.

Typical schemes of UT

a) one axis through testing  b) angle testing

c) one surface testing  d) curve surface testing
CHAPTER (7)
Measurement of Surface Texture
**Terminology on Surface Texture**

- **Flaw (defect):** random irregularities such as scratches, cracks, holes, tears, inclusions, etc.
- **Lay (directionality):** direction of the predominant surface pattern (see below for various lays).
- **Waviness:** recurrent deviation from a flat surface.
- **Roughness:** closely spaced irregular deviations on a scale smaller than that of waviness.
- **Surface texture (topography):** refers to primary (form), waviness and roughness profiles.
- **Surface finish:** refers to only roughness profile (ignoring the shape and underlying waviness).

![Diagram showing surface features and lay symbols]

<table>
<thead>
<tr>
<th>Lay symbol</th>
<th>Surface pattern</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>=</td>
<td></td>
<td>Lay is parallel to line representing surface to which symbol is applied.</td>
</tr>
<tr>
<td>⊥</td>
<td></td>
<td>Lay is perpendicular to line representing surface to which symbol is applied.</td>
</tr>
<tr>
<td>X</td>
<td></td>
<td>Lay is angular in both directions to line representing surface to which symbol is applied.</td>
</tr>
<tr>
<td>M</td>
<td></td>
<td>Lay is multidirectional.</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>Lay is circular relative to center of surface to which symbol is applied.</td>
</tr>
<tr>
<td>R</td>
<td></td>
<td>Lay is approximately radial relative to the center of the surface to which symbol is applied.</td>
</tr>
<tr>
<td>P</td>
<td></td>
<td>Lay is particulate, nondirectional, or protuberant.</td>
</tr>
</tbody>
</table>
Measurement of Surface Texture

> Surface texture is measured using a profilometer (roughness tester) which consists of a stylus (tracing probe) with a perfectly sharp tip made of hard material (e.g. diamond).

> The stylus is set vertically in a way that the tip of stylus will be in contact with the surface.

> Then, the stylus is moved horizontally along the surface to be measured in order to follow the surface contours where the stylus path is smoother than the actual path.

> Various types of stylus are available for different applications.
Profilometers (Roughness Testers)

The profilometers are classified as with or without skid:

- **Profilometer with skidded gage:** In skidded gages, the sensitive diamond-tipped stylus is contained within a probe, which has a skid that rests on the workpiece. Thus, skidded gages use the workpiece itself as the reference surface to measure roughness only.

- **Skidless gage profilometer:** Skidless gages use an internal precision surface as a reference. This enables skidless gages to be used not only for roughness, but also waviness and form profiles.
Typical Measurements

Dimension, form and texture can be measured at once over curved or straight surfaces

The measured profiles can be evaluated using dedicated software to suppress roughness and waviness profiles

Measurement of ball tracks and ring grooves using skidless tracing arms

Measurement of inner surfaces of gears
Surface Texture Parameters

The parameters related with measurement of surface texture are divided into three groups. Table shows the parameters for primary, waviness and roughness measurements:

<table>
<thead>
<tr>
<th>Parameter Group</th>
<th>Explanation</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>The vertical characteristics of the surface deviations</td>
<td>Pa, Pq, Pv, Pp, Pt, Psk, Pku, Pz</td>
</tr>
<tr>
<td>Spacing Parameters</td>
<td>The horizontal characteristics of the surface deviations</td>
<td>Psm</td>
</tr>
<tr>
<td>Hybrid Parameters</td>
<td>Combination of both vertical and horizontal characteristics of the surface deviations</td>
<td>PΔq, Pλq</td>
</tr>
</tbody>
</table>
Assessment of Roughness Profile

The profile shown below is a typical 2D roughness profile:

> **Assessment (evaluation) length \( (L) \):** Length used for assessing the profile for roughness measurement. For this length, at least five consecutive lengths are taken as standard.

> **Sampling length \( (l) \):** The profile mean line is determined and the profile is divided into equal sampling lengths (from \( l_1 \) to \( l_5 \)).

> **Cut-off length \( (\lambda_C) \):** A filter to remove or reduce unwanted data to look at wavelengths in the assessment region. Sampling length is also known as cut-off length.
Roughness Parameters – $R_a$ and $R_q$

> **Roughness Average ($R_a$):** Universally recognised and commonly used roughness parameter, which is the arithmetic mean of departures from the mean line. It is also known as Center Line Average (CLA) or Arithmetic Average (AA).

> **Root Mean Square (RMS) Roughness ($R_q$):** It is the RMS average of roughness profile ordinates.

> $R_a$ is a very stable and repeatable parameter, which makes it good for random type surfaces. However, it cannot provide distinction between peaks and valleys.

> $R_q$ is more sensitive to peaks and valleys due to the reason that the amplitudes are squared.
Misinterpretation of Surface Roughness based on $R_a$

> As said before, it is not possible to make a distinction between peaks and valleys by using $R_a$.

> Three roughness profiles shown below have the same $R_a$ value although they seem to be different. Therefore, the assessment of these profiles using $R_a$ will cause inaccurate conclusions to be made.

> For this purpose, there is need for more specific and sensitive roughness parameters in order to make a more reliable assessment.
Roughness Parameters – $R_z$ and $R_{z_{\text{max}}}$

- **Mean Roughness Height/Depth ($R_z$):** The mean of roughness heights/depths at each sample length.

- **Maximum Roughness ($R_{z_{\text{max}}}$):** The largest of five roughness heights/depths at each sample length.

- $R_z$ is more sensitive than $R_a$ to changes on the surface as the maximum profile heights are examined rather than average of peak and valleys. In addition, $R_{z_{\text{max}}}$ is useful for surfaces where a single defect is not permissible (e.g. a seal with a scratch).

- $R_z$ and $R_{z_{\text{max}}}$ are used together to monitor the variations of surface finish in a production process. Similar values of them indicate a consistent surface finish, while a significant difference between them indicates a surface defect in an otherwise consistent surface.

\[
R_{z_{\text{max}}} = \left(R_{z_i}\right)_{\text{max}}
\]

\[
R_z = \frac{1}{n} \sum_{i=1}^{n} R_{z_i}
\]
Roughness Parameters – \( R_p, R_v, R_t \)

> **Maximum Height** (\( R_p \)): The maximum roughness height (peak) within each sampling length.

> **Maximum Depth** (\( R_v \)): The maximum roughness depth (valley) within each sampling length.

> **Mean Levelling** (\( R_{pm} \)): The mean of five consecutive heights (peaks) from each sampling length.

> **Peak-to-Valley Roughness** (\( R_t \)): The largest peak-to-valley in the entire profile.

> \( R_{pm} \) is recommended for bearing and sliding surfaces and surface substrates prior to coating.

> A low value of \( R_{pm} \) and a large value of \( R_z \) indicates a plateau surface.

> The ratio of \( R_{pm} / R_z \) quantifies the asymmetry of profile.

> \( R_v \) is a good parameter where stress is a major factor whereas \( R_p \) is to control coating quality.
**Spacing Parameter – \( R_{sm} \)**

> **Mean Spacing (\( R_{sm} \)):** The mean spacing of \( S_1, S_2, \ldots, S_n \) between profile peaks as they pass through the mean line (spacing is the distance between points that cross the mean line in an upward direction).

\[
S_1 \quad S_2 \quad S_3
\]

> **\( R_z \) to \( R_a \) Conversion:** Based on BS 1134/1-1972 standard, \( R_z = x_4 - x_7 \) \( R_a \). The actual ratio depends upon the shape of the profile.

Selection of Cutt-off Length

> Changing cut-off value (which changes amount of "averaging" and "smoothing") can have huge impact on measurement of roughness and waviness.

> Choosing smaller cut-off lengths will result in smaller roughness values even though the real surface could be very rough. The picture shown presents the same surface with different cut-offs. The profile on the left gives twice the $R_a$ value of the profile on the right.

> Thus, there are recommended values for choosing the appropriate cut-offs, which were defined by ISO 4288-1996.

<table>
<thead>
<tr>
<th>Periodic Profiles</th>
<th>Non-Periodic Profiles</th>
<th>Cut-offs</th>
<th>Evaluation Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_m$ (mm)</td>
<td>$R_z$ (µm)</td>
<td>$R_a$ (µm)</td>
<td>$\lambda_C$ (mm)</td>
</tr>
<tr>
<td>&gt;0.013 to 0.04</td>
<td>(0.025) to 0.1</td>
<td>(0.006) to 0.02</td>
<td>0.08</td>
</tr>
<tr>
<td>&gt;0.04 to 0.13</td>
<td>&gt;0.1 to 0.5</td>
<td>&gt;0.02 to 0.1</td>
<td>0.25</td>
</tr>
<tr>
<td>&gt;0.13 to 0.4</td>
<td>&gt;0.5 to 10</td>
<td>&gt;0.1 to 2</td>
<td>0.8</td>
</tr>
<tr>
<td>&gt;0.4 to 1.3</td>
<td>&gt;10 to 50</td>
<td>&gt;2 to 10</td>
<td>2.5</td>
</tr>
<tr>
<td>&gt;1.3 to 4</td>
<td>&gt;50 to 200</td>
<td>&gt;10 to 80</td>
<td>8</td>
</tr>
</tbody>
</table>
## ISO Standards on Surface Texture

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISO 1302 - 2001</td>
<td>Indication of Surface Texture</td>
</tr>
<tr>
<td>ISO 3274 - 1996</td>
<td>Nominal Characteristics of Contact (Stylus) Instruments</td>
</tr>
<tr>
<td>ISO 4287 - 1997</td>
<td>Terms, Definition and Surface Texture Parameters</td>
</tr>
<tr>
<td>ISO 4288 - 1996</td>
<td>Rules and Procedures for Assessment of Surface Texture</td>
</tr>
<tr>
<td>ISO 5436-1 - 2000</td>
<td>Calibration, Measurement Standards</td>
</tr>
<tr>
<td>ISO 5436-2 - 2000</td>
<td>Calibration, Soft Gages</td>
</tr>
<tr>
<td>ISO 8785 - 1999</td>
<td>Surface Imperfections - Terms, Definitions and Parameters</td>
</tr>
<tr>
<td>ISO 11562 - 1996</td>
<td>Metrological Characteristics of Phase Correct Filters</td>
</tr>
<tr>
<td>ISO 12085 - 1996</td>
<td>Motif Parameters</td>
</tr>
<tr>
<td>ISO 12179 - 2000</td>
<td>Calibration of Contact (Stylus) Instruments</td>
</tr>
<tr>
<td>ISO 13565 - 1996</td>
<td>Characterization of Surfaces Having Stratified Functional Properties</td>
</tr>
</tbody>
</table>
Surface Finish Tolerances in Manufacturing

METAL CUTTING
- sawing
- planing, shaping
- drilling
- milling
- boring, turning
- broaching
- reaming

ABRASIVE
- grinding
- barrel finishing
- honing
- electro-polishing
- electrolytic grinding
- polishing
- lapping
- superfinishing

CASTING
- sand casting
- perm mold casting
- casting investment
- casting
- die casting

FORMING
- hot rolling
- forging
- cold rolling
- drawing
- roller burnishing

OTHER
- flame cutting
- chemical milling
- electron beam cutting
- laser cutting
- EDM

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CHAPTER (8)
Measurement of Screw Threads
> **Thread**: the helical grooves opened to inner and outer surfaces.

> **External thread (screw)**: A thread on the external surface of a cylinder.

> **Internal thread (nut)**: A thread on the internal surface of a cylinder.

> **Major diameter (diş üstü çap)**: The largest diameter of a screw thread.

> **Minor diameter (diş dibi çap)**: The smallest diameter of a screw thread.

> **Pitch diameter (bölüm çapı)**: The diameter of an imaginary cylinder, the surface of which cuts the thread forms where the width of the thread and groove are equal.

> **Crest**: The edge or surface that joins the sides of a thread and is farthest from the cylinder or cone from which the thread projects.

> **Root**: The edge or surface that joins the sides of adjacent thread forms and coincides with the cylinder or cone from which the thread projects.

> **Thread Depth**: The distance between crest and root.

> **Pitch (hatve, adım)**: The distance between corresponding points on adjacent thread forms measured parallel to the axis.> **Right-hand thread**: A thread that when viewed axially winds in a CW and receding direction. Threads are RH unless otherwise specified.

> **Left-hand thread**: A thread that when viewed axially winds in a CCW and receding direction. These threads are designated as LH.
Multiple Threads

> **Lead:** The distance that a threaded part moves axially in one complete revolution.

> **Lead or Helix Angle:** The angle made by the pitch helix, defined as: \( \lambda = \arctan \left( \frac{\text{Lead}}{\pi \times \text{Pitch Dia.}} \right) \)

> **Single thread:** A thread having the form produced on only one helix of cylinder. On a single thread, the lead and pitch are equivalent. Threads are always single unless otherwise specified.

> **Multiple thread:** A thread combination having the same form produced on two or more helices where the lead is an integral multiple of the pitch (e.g. on a double thread, lead is twice the pitch). A multiple thread permits a more rapid advance without a coarser (larger) thread form.
Thread Types and Designation

> There are several thread forms used for specific applications. Selection of the appropriate thread form depends upon the functionality, size and purpose of the required job.

> The threads are designated in Metric or British system as shown below.

**British System**

.250-20 UNC-2A-LH

- **a**: Major diameter (inch)
- **b**: Threads per inch
- **c**: Form (i.e. Unified National Coarse)
- **d**: External thread (B for internal)
- **e**: Left-hand thread (RH for right-hand)

**Metric System**

M20 x 2

- **x**: Metric screw thread
- **y**: Major diameter (mm)
- **z**: Pitch (mm)

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The threads can be measured (checked) using fixed limit gauges such as **adjustable thread ring and plug gauges**, which provide **GO (green)** or **NOT-GO (red)** type of measurement.

**Screw pitch gauge** (consisting of a metal case having several leaves) can also be used. Each leaf has teeth corresponding to a definite pitch. By matching these teeth with threads on work, the correct pitch can be read directly from the leaf.

**Thread rolls** (with various forms and known dimensions) are used to measure (check) internal threads.
Thread Micrometers

Screw thread micrometers having various type and size of interchangeable spindle and anvils are used for measuring the pitch diameter of external threads.

The pitch diameter of internal threads are measured by inside thread micrometers with various type and dimensions for different applications.
Three-Wire Method

> **Three-wire method** is one of the most accurate and versatile ways of measuring the **pitch diameter** of a thread by using three lapped and polished wires and a micrometer.

> Wires touching the threads at the pitch diameter are “**Best Size Wires**”. Such wires are used since the measurements are least affected by errors that may be present in the angle of the thread.

![Diagram of thread with wires](image)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_i$</td>
<td>Minor diameter</td>
</tr>
<tr>
<td>$D_p$</td>
<td>Pitch diameter</td>
</tr>
<tr>
<td>$D_o$</td>
<td>Major diameter</td>
</tr>
<tr>
<td>$h$</td>
<td>Depth of thread</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Lead (helix) angle</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Flank angle</td>
</tr>
<tr>
<td>$p$</td>
<td>Pitch</td>
</tr>
<tr>
<td>$D_w$</td>
<td>Wire diameter</td>
</tr>
<tr>
<td>$H_w$</td>
<td>Measurement over wires</td>
</tr>
</tbody>
</table>
Three-Wire Method

> The table gives the pitch diameter for some thread types having lead angle from 0° to 5.

<table>
<thead>
<tr>
<th>Thread Type</th>
<th>Thread Angle ((2\alpha))</th>
<th>Thread Depth ((h))</th>
<th>Wire Size ((D_w))^a</th>
<th>Measurement over wires ((H_w))</th>
<th>Pitch Diameter ((D_p))^b,c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unified National</td>
<td>60°</td>
<td>0.649519 (p)</td>
<td>0.57735 (p)</td>
<td>(D_o - 1.51555 (p) + 3 (D_w))</td>
<td>(H_w - (3 (D_w) - 0.86603 (p)))</td>
</tr>
<tr>
<td>American National</td>
<td>60°</td>
<td>0.8 (p)</td>
<td>0.57735 (p)</td>
<td>((D_o - 0.8660254 (p) + 3.00049 (D_w))/1.00049)</td>
<td>1.00049 (H_w - (3.00049 (D_w) - 0.86603 (p)))</td>
</tr>
<tr>
<td>Sharp V</td>
<td>60°</td>
<td>0.8660254 (p)</td>
<td>0.57735 (p)</td>
<td>(D_o - 1.73205 (p) + 3 (D_w))</td>
<td>(H_w - (3 (D_w) - 0.86603 (p)))</td>
</tr>
<tr>
<td>Metric</td>
<td>60°</td>
<td>0.649519 (p)</td>
<td>0.57735 (p)</td>
<td>(D_o - 1.51553 (p) + 3 (D_w))</td>
<td>(H_w - (3 (D_w) - 0.86603 (p)))</td>
</tr>
<tr>
<td>Whitworth</td>
<td>55°</td>
<td>0.64033 (p)</td>
<td>0.56369 (p)</td>
<td>(D_o - 1.60082 (p) + 3.16568 (D_w))</td>
<td>(H_w - (3.16568 (D_w) - 0.96049 (p)))</td>
</tr>
<tr>
<td>Acme</td>
<td>29°</td>
<td>0.5 (p)</td>
<td>0.51645 (p)</td>
<td>(D_o - 2.43334 (p) + 4.9939 (D_w))</td>
<td>(H_w - (4.9939 (D_w) - 1.933357 (p)))</td>
</tr>
</tbody>
</table>

^a The general formula is: \(D_w = 0.5 \sec(\alpha) \(p\)\)

^b The general formula is: \(D_p = H_w - [D_w (1 + \csc(\alpha)) - 0.5 \(p\) \cot(\alpha)]\)

^c For tapered threads, the taper angle \((\beta)\) is used: \(D_p = H_w - [D_w (1 + \csc(\alpha)) - 0.5 \(p\) (\cot(\alpha) - \tan^2(\beta) \tan(\alpha))]\)
## Comparison of Measurement Methods

<table>
<thead>
<tr>
<th>METHOD</th>
<th>ADVANTAGES</th>
<th>DISADVANTAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thread Gauges &amp;</td>
<td>☀ Inspects the complete thread profile</td>
<td>☎ Reveals only if the thread is correct or incorrect (i.e. it gives no</td>
</tr>
<tr>
<td>Thread Rolls</td>
<td>☀ Simple to use with minimum training</td>
<td>information related to its tolerance)</td>
</tr>
<tr>
<td></td>
<td>☀ The inspected thread can be judged correct or</td>
<td>☎ Time consuming when setting up and controlling the process, and difficult</td>
</tr>
<tr>
<td></td>
<td>incorrect by assuming use of both GO and NOT-GO</td>
<td>and/or expensive to calibrate</td>
</tr>
<tr>
<td>Thread Micrometer</td>
<td>☀ Accurate if the flank angle is correct</td>
<td>☎ Manufacturing tolerances and wear allowances on the gauge give reduced</td>
</tr>
<tr>
<td></td>
<td>☀ Can be used on most thread types with the</td>
<td>tolerances on the thread</td>
</tr>
<tr>
<td></td>
<td>same flank angle</td>
<td>☎ Only suitable for the specific thread denomination (tolerance) stated on</td>
</tr>
<tr>
<td></td>
<td>☀ Suitable for machine set-up and process control</td>
<td>the gauge</td>
</tr>
<tr>
<td>Three-Wire System</td>
<td>☀ Very accurate results if the flank angle and</td>
<td>☎ Requires calculation to find the correct measurement result</td>
</tr>
<tr>
<td></td>
<td>pitch of threads are correct</td>
<td>☎ Measures only pitch diameter</td>
</tr>
<tr>
<td></td>
<td>☀ Can be used for almost all thread types</td>
<td>☎ Requires special (and thus costly) micrometer</td>
</tr>
<tr>
<td></td>
<td>☀ Suitable for machine set-up and process control</td>
<td>☎ Can be used on the most thread types with the same flank angle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>☎ Requires special (and thus costly) micrometer</td>
</tr>
</tbody>
</table>
CHAPTER (9)
Measurement of Gears
Gear Tooth Terminology

Root Diameter : Diameter of root circle
Pitch Diameter : Diameter of imaginary pitch circle specifying addendum and dedendum
Outside Diameter : Diameter of addendum (outside) circle
Addendum : Radial distance from pitch to top of tooth
Dedendum : Radial distance from pitch to bottom of tooth
Circular Pitch : Distance on pitch circle from a point on one tooth to corresponding point on the adjacent tooth
Tooth Thickness : Thickness of a tooth along the pitch circle
Clearance : Distance between top of a tooth and bottom of mating space
Working Depth : Distance a tooth projects into mating space
Whole Depth: Total height of the tooth
Face width: The width of the tooth
Gear Meshing

> Two gears having an identical module can be meshed. Meshing of two spur gears with a center distance is shown below.

> The line of action (also known as “pressure line”) is a line that is drawn tangent to the base circle of pinion and gear.

> When two gear teeth are in contact, the kinematic principle of gearing is demonstrated: angular velocity ratio of meshing gears is constant along the line of action.

> The pressure angle is the angle between the tangent to the pitch circles and the line of action.

> Gear catalogs are classified according to number of teeth and pressure angle.
Tooth Thickness Measurement

The measurement of tooth thickness is a common feature for gears to be checked using following methods (see the standards by American Gear Manufacturers Association – www.agma.org):

> **Measurement over Pins:** Special size pins are placed between the teeth and the distance between pins are measured (AGMA 231.52 and AGMA 239.03 for spur and helical gears).

> **Vernier Gear Tooth Caliper:** The chordal tooth thickness and height are measured.

> **Span Measurement:** The distance between a number of teeth is measured (AGMA 239.01)
Measurement using Vernier Gear Tooth Caliper

The chordal thickness (X) and height (Y) are:

\[ X = 2R_p \sin \theta \quad \text{and} \quad Y = a + R_p (1 - \cos \theta) \]

where \[ a = m = \frac{2R_p}{z} \quad \text{and} \quad \theta = \frac{\pi}{2z} = 90^\circ \]

If a large number of gears for a set (having different values of \( z \)) are to be measured, calculations would become laborious. Hence, constant chord thickness (\( X_c \)) and height (\( Y_c \)), which are independent of \( z \), are calculated:

\[ X_c = \frac{\pi m}{4} \cos^2 \psi \quad \text{and} \quad Y_c = m - \frac{\pi m}{8} \sin 2\psi \]

**Constant chord** is defined as the chord joining points on opposite faces of tooth that make contact with the mating teeth at the line of action. It should be noted that formulas above are for spur gears. For **helical gears**, normal module (\( m_n \)) must be employed in calculations.

---

\( a \) : addendum  
\( R_p \) : pitch radius  
\( \Psi \) : pressure angle  
\( m \) : module  
\( z \) : number of teeth
Measurement over Pins

The **tooth thickness** is found as:

\[ t = D_p \left( \text{inv } e - \text{inv } \psi - \frac{D_g}{D_p \cdot \cos \psi} + \frac{\pi}{z} \right) \]

The **measurement over pins** equals to:

For z is even: \( H_e = 2h + D_g \)
For z is odd: \( H_o = 2h \cdot \cos \left(90/\pi \right) + D_g \)

The **involute of angle** \( e \) is calculated by:

\[ \text{inv } e = \tan e - \left( e \times \frac{\pi}{180} \right) \]

The **angle** \( e \) is calculated as:

For z is even: \( e = \arccos \left( \frac{z \cdot \cos \psi}{D_p \left( H_e - D_g \right)} \right) \)

For z is odd: \( e = \arccos \left( \frac{z \cdot \cos \psi}{D_p \left( H_o - D_g \right)} \cdot \cos \left( \frac{90}{z} \right) \right) \)

**Pin diameter** (for internal gears): \( D_g = 13.58 \, m \)

**Pin diameter** (for external gears): \( D_g = 13.97 \, m \)
Span Measurement

The distance $M$ is expressed as:

$$M = D_p \cdot \cos\psi \cdot \left( \frac{\pi}{2z} + \text{inv} \psi + \frac{\pi}{z} \right)$$

Thus, the tooth thickness can be obtained based on base circle pitch and measured distance of $M$.

It should be noted that **module in normal plane** ($m_n$) and **pressure angle in normal plane** ($\psi_n$) must be employed in case of helical gears.

**Symbols**:
- $M$: span distance
- $D_p$: pitch diameter
- $\psi$: pressure angle
- $z$: number of teeth
- $s$: number of tooth spaces within $M$
- $P_b$: base circle pitch
- $t_b$: tooth thickness at base circle
<table>
<thead>
<tr>
<th>METHOD</th>
<th>ADVANTAGES</th>
<th>DISADVANTAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement over Pins</td>
<td>☺ Measurements are not affected by outside diameter variations or by runout of the outside diameter.</td>
<td>☎ It is crucial that the most appropriate pins must be selected. For gears having non-standard features, the estimate of pin diameter may be a bottleneck.</td>
</tr>
<tr>
<td>Vernier Gear Tooth Caliper</td>
<td>☺ It is relatively easier to use and cheaper as compared with other methods.</td>
<td>☎ The precision of vernier caliper directly affects the measurements.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>☎ The measurements depend on two vernier readings, each of which is a function of each other.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>☎ Measurement is made with an “edge” of the measuring jaw (not its “face”), which again does not lend itself to accurate measurement.</td>
</tr>
<tr>
<td>Span Measurement</td>
<td>☺ Measurements are not affected by outside diameter variations or by runout of the outside diameter.</td>
<td>☎ It cannot be applied when a combination of high helix angle and narrow face width prevent the caliper from spanning a sufficient number of teeth.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>☎ Readings are influenced by errors in base pitch and tooth profile. Readings would be erroneous if attempted on a portion of profile which had been modified from true involute shape.</td>
</tr>
</tbody>
</table>
Gear Tooth Profile Measurements using CMM

Profile trace of poorly molded gear:

- G

Properly molded gear:

- CMM measurements

Engineering Metrology - Professor Nabil Gadallah
CHAPTER (10) Limits and Limit Gauges
Various Types of Limit Gauges

Cylindrical Pin Gauge

Adjustable Snap Gauge

Plain Ring Gauge

Thread Plug Gauge

Cylindrical Plug Gauge

Adjustable Thread Ring Gauge

Progressive Cylindrical Plug Gauge
Taylor’s Theory of Gauging

> This theory is the key to design of limit gauges, and thus it defines the function (i.e. the form) of most limit gauges.

> **GO gauge** checks the **Maximum Material Condition (MMC)**, and it should check as many dimensions as possible.

> **NOT-GO gauge** checks the **Least Material Condition (LMC)**, and it should only check one dimension.

> **GO gauge** is used to ensure that the **MMC** is not exceeded. Thus, it should be made to the MMC based on wear and gauge tolerances which are explained in the following slides.

> However, if **NOT-GO gauge** is made to both dimensions of LMC, a condition would arise where the width of work is within specified limits whereas the length is oversize. Such a gauge will not enter the work, and therefore the work will be accepted although the length is outside the limits (see the figure). **Thus, a separate NOT-GO gauge is required for each individual dimension.**

> **NOT-GO gauge** is always relatively shorter, and approximately equal in length to the hole diameter. **GO gauge should be equal in length to about three or four diameters.** This would enable to check that the MMC is not exceeded due to **geometric errors** (e.g. straightness as illustrated in figure).
Tolerances in Limit Gauges

- For a hole to be checked; GO gauge is a cylinder with diameter equal to the **minimum hole size** whereas NOT-GO gauge is a cylinder with diameter equal to the **maximum hole size**.
- However, this is not as simple as it is due to the fact that the limits of size are required for the work which states that **nothing can be made to an exact size including also the gauges**.
- The gauge maker needs a tolerance to which the gauges may work. Defining this **gauge tolerance relative to the nominal gauge size** is critical.
- For instance; if the gauge tolerance increases the size of GO gauge and decreases the size of NOT-GO gauge, then the gauge will **tend to reject good work** which is near the upper or lower size of limits.
- If the otherwise happens, the gauge will **tend to accept doubtful work** which is just outside the specified limits.

> This decision has been made in B.S. 969 as follows:

1) The tolerance on GO gauge shall be **within** the work tolerance zone.
2) The tolerance on NOT-GO gauge shall be **outside** the work tolerance zone.

- Allowance must also be made for the initial wear which takes place on a new gauge. Thus, the tolerances shall be:
  1) Gauge tolerance = 10% of work tolerance
  2) Wear allowance = 5% of work tolerance
Tolerancing System and Wear Allowance

> There are two systems for gauge tolerances: unilateral and bilateral

> In unilateral system; the gauge tolerance zones lie entirely within the work tolerance zone. The disadvantage of this system is that certain parts may be rejected as if they were outside the limits.

> In bilateral system; the gauge tolerance zones are bisected by high and low limits of work tolerance zone. The disadvantage is that parts within the working limits can be rejected and parts outside the working limits can be accepted.

> In modern limit systems, unilateral system is preferred.

> Measuring surfaces of GO gauges constantly rub against the surfaces of parts in inspection, thus they loose their initial size due to wear.

> Therefore, wear allowance (usually 5% of work tolerance) is added to nominal dimension of GO plug gauge to prolong the service life of gauge.

> Wear allowance is not considered for NOT-GO gauge as it is not subjected to so much wear.

> Size of GO plug gauge is reduced while that of GO snap gage increases. Therefore, the wear allowance is subtracted from the nominal dimension of GO snap gauge.

![Diagram of Tolerancing Systems](image-url)
An Example

Suppose that size of the hole to be tested is: 25 ± 0.02 mm

- Highest limit of hole = 25.02 mm
- Lowest limit of hole = 24.98 mm
- Work tolerance = 0.04 mm
- Gauge tolerance = 10% of work tolerance = 0.004 mm
- Wear allowance = 5% of work tolerance = 0.002 mm
- Nominal size of GO plug gauge = 24.98 + 0.002 = 24.982 mm

If bilateral system was used, the gauge dimensions would be: 24.982 ± 0.002 mm and 25.02 ± 0.002 mm

For shaft measurement using a snap gauge, the order of tolerances between must be reversed:

- Nominal size of GO snap gauge = 25.02 - 0.002 = 25.018 mm
- Dimension of GO snap gauge: 25.018 ± 0.004 mm
- Dimension of NOT-GO snap gauge: 24.98 ± 0.004 mm
Pros and Cons of Limit Gauges

The advantages of limit gauges can be summarised as follows:

😊 Limit gauges are conveniently used in mass production for controlling various dimensions.
😊 They can easily be used by semi-skilled people.
😊 They are economical in their own cost as well as engaging cost.

The followings are the limitations of limit gauges:

➊ Do not indicate the actual size of the component
➋ Susceptible for wear, expansion and collapse
➌ Large number of gauges is required necessitating larger space
➍ Cannot handle finer quality jobs due to precision issues
➎ Require frequent checking of gauge dimensions